

Test exercises for the introductory seminar

Alexander Malkis

Out: 29.06.2016. Deadline: 06.07.2016.

Problems 1–4 have been solved in class.

Problem 5:

Is $2^{2017} - 1$ prime? Provide a proof of primality or a nontrivial factorization.

Problem 6:

Are there infinitely many natural numbers a such that for all natural numbers n the number $n^4 + a$ is never prime?

Problem 7:

Let p be a prime number. Provide a simple necessary and sufficient condition on p for the existence of $p - 1$ consecutive natural numbers whose sum divides the sum of their squares.

Problem 8:

Prove or refute:

- (i) If a prime is divided by 30, the remainder is also a prime.
- (ii) If a prime is divided by 60, the remainder is also a prime.